

Subject: Mathematics.

Q1- (i)solution

$$\frac{5}{2} + \frac{10}{3} \div \frac{7}{4}$$

$$= \frac{15+20}{6} \times \frac{4}{7} = \frac{35}{6} \times \frac{4}{7} = \frac{5}{3} \times 2$$

$$= \frac{10}{3} \text{ Ans (2mks)}$$

(ii)solution

$$9x(x+1)=4$$

$$9x^2 + 9x - 4 = 0$$

$$(3x+1)(3x-4)=0$$

$$\text{Either } 3x+1=0 \text{ or } 3x-4=0$$

$$x=-\frac{1}{3} \text{ or } x=\frac{4}{3} \text{ Ans (2mks)}$$

Q2-(a) solution

$$\log 4 / \log 8 = \log 2^2 / \log 2^3$$

$$= 2 \log 2 / 3 \log 2 = \frac{2}{3} \text{ Ans (2mks)}$$

(b)solution

$$3\sqrt{12} - 2\sqrt{3}$$

$$= 3\sqrt{4} \times \sqrt{3} - 2\sqrt{3} = 6\sqrt{3} - 2\sqrt{3}$$

$$= 4\sqrt{3} \text{ Ans (2mks)}$$

Q3-(a)-solution:

$$\sin \alpha = 0.5 (16/81)^{-3/4} = (3/2)^{4 \times 3/4}$$

$$\alpha = \sin^{-1} 0.5 = 30^\circ, 150^\circ \text{ Ans (2mks)}$$

(b)-Solution:

$$= (3/2)^3 = \frac{27}{8} \text{ Ans (1mks)}$$

(c)Solution

$$\log_x 8 = 3$$

$$x^3 = 8 \quad x^3 = 2^3 \quad x = 2 \text{ Ans (1mk)}$$

Q4-(a)Solution:

$$x/2 - 2 \geq 2x = 8$$

$$x - 4 \geq 4x + 16$$

(b)Solution

$$(0.0315)^2 - (0.0185)^2$$

$$= (0.0315 + 0.0185)(0.0315 - 0.0185)$$

$$X - 4X \geq 16 + 4 \quad = 0.05 \times 0.013 \quad = 0.00065$$

$$-3X \geq 20 \quad \approx 0.001 \text{ Ans (2mks)}$$

$$X \leq -20/3 \text{ Ans (2mks)}$$

Q5-(a) Solution:

$$\begin{pmatrix} 3 & -1 \\ 20 & y \end{pmatrix} \begin{pmatrix} X \\ -1 \end{pmatrix} = \begin{pmatrix} 7 \\ 20 \end{pmatrix} \quad \text{Substituting } X = 2 \text{ in equation (ii)}$$

$$\begin{pmatrix} 3x+1 \\ 20X-y \end{pmatrix} = \begin{pmatrix} 7 \\ 20 \end{pmatrix} \quad 20X - y = 20$$

$$3X + 1 = 7 \quad \text{(i)} \quad 40 - y = 20$$

$$3X = 7 - 1 = 6 \quad x = 6/3 = 2 \quad X = 2, y = 20 \text{ Ans (2mks)}$$

Q6-(i) Solution (ii) Solution

$$P + q = \begin{pmatrix} 2 & 4 \\ 3 & 6 \end{pmatrix} + \begin{pmatrix} -2 & 5 \\ 6 & 1 \end{pmatrix} Pq = \begin{pmatrix} 2 & 4 \\ 3 & 6 \end{pmatrix} \begin{pmatrix} -2 & 5 \\ 6 & 1 \end{pmatrix}$$

$$P + q = \begin{pmatrix} 0 & 9 \\ 9 & 7 \end{pmatrix} \text{ Ans (2mks)} \quad = \begin{pmatrix} -4+24 & 10+4 \\ -6+36 & 15+6 \end{pmatrix}$$

$$Pq = \begin{pmatrix} 20 & 14 \\ 30 & 21 \end{pmatrix} \text{ Ans (2mks)}$$

Q7-(a) Solution (b) Solutin:

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{the Slope of AB} = Y_2 - Y_1 / X_2 - X_1$$

$$= \sqrt{(3+1)^2 + (4-2)^2} = 4 - 2/3 + 1 = 2/4 = 1/2$$

$$= \sqrt{4^2 + 2^2} = \sqrt{16+4} \quad \text{the slope of AB} = 1/2 \text{ Ans (2mks)}$$

$$= \sqrt{20}$$

Length AB = $2\sqrt{5}$ units Ans (2mks)

Q9-(a)Solution

$$X+Y=8 \text{-----(i)}$$

$$XY=15 \text{-----(ii)}$$

$$Y=8-X$$

Substituting $Y=8-X$ in equaton (ii)

$$X(8-X)=15$$

$$8X - X^2 = 15$$

$$X^2 - 8X + 15 = 0$$

$$(X-3)(X-5) = 0$$

Either $X-3=0$ or $X-5=0$

$$X=3 \text{ or } X=5 \text{ Ans (2mks)}$$

(b)Solution

$$a=-3, b=1 \text{ and } r=5$$

$$\text{Using } (X-a)^2 + (Y-b)^2 = r^2$$

$$(X+3)^2 + (Y-1)^2 = 5^2$$

$$X^2 + 6X + 9 + Y^2 - 2Y + 1 = 25$$

$$X^2 + Y^2 + 6X - 2Y + 10 - 25 = 0$$

The required equation of the circle is

$$X^2 + Y^2 + 6X - 2Y - 15 = 0 \text{ Ans (2mks)}$$

Q9-(i)Solution

Let $S=\{1,2,3,4,5,6\}$ and $A=\{1,2,3,4\}$

$$\text{Using } P(A) = n(A)/n(S) = 4/6 = 2/3$$

$$P(A) = 2/3 \text{ Ans (2mks)}$$

(ii)Solution

Let $S=\{1,2,3,4,5,6\}$

$$A=\{1,3,5\}$$

$$B=\{4,6\}$$

$$\text{Using } P(A \cup B) = n(A)/n(S) + n(B)/n(S)$$

$$=3/6 + 2/6 = 3 + 2/6 = 5/6$$

$$P(A \cup B) = 5/6 \text{ Ans (2mks)}$$

Q10-(i)Solution:

Area of trapezium = $1/2 \times \text{height} \times \text{sum of two parallel sides}$

$$= 1/2 \times 7 \times (18+14)$$

$$= 1/2 \times 7 \times 32 = 7 \times 16 = 112$$

(ii)Solution:

$$(1+x)^5 = 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$$

$$\text{Ans (2mks)}$$

$$\text{Area of the trapezium} = 112 \text{ cm}^2 \text{ Ans (2mks)}$$

SECTION (B) (60 MARKS)

Given that $f(x) = 3-2x$, $g(x) = x^2+1$

Q11-(i) Solution:

$$[f+g](2) = f(2) + g(2)$$

$$= (3-4) + (4+1)$$

$$= -1+5 = 4$$

$$[f+g](2) = 4 \text{ ans (2mks)}$$

(iii) Solution:

$$F[g(2)] = g(2) = 2^2+1 = 4+1 = 5$$

$$F[g(2)] = f(5) = 3 - (2 \times 5) = 3 - 10 = -7$$

$$F[g(2)] = -7 \text{ Ans (3mks)}$$

(ii) Solution

$$[f-g](2) = f(2) - g(2)$$

$$= (3-4) - (4+1)$$

$$= -1-5 = -6$$

$$[f-g](2) = -6 \text{ Ans (2mks)}$$

(iv) Solution $f(2) = 3-2(2) = 3-4 = -1$

$$g[f(2)] = g(-1) = (-1)^2+1 = 1+1 = 2$$

$$g[f(2)] = 2 \text{ ans (3mks)}$$

(v) Solution:

$$g'(x) = x^2+1$$

$$g'(x) = 2x \text{ Ans (2mks)}$$

Q12-(a) Solution:

| height | Frequency | X | fX |
|--------|---------------|----|-----------------|
| 0-2 | 10 | 1 | 10 |
| 3-5 | 9 | 4 | 36 |
| 6-8 | 6 | 7 | 42 |
| 9-11 | 4 | 10 | 40 |
| 12-14 | 1 | 13 | 13 |
| | $\sum f = 30$ | | $\sum fx = 141$ |

Using $\bar{x} = \sum fx / \sum f = 141/30 = 4.7\text{cm}$ Ans (6mks)

(b) Solution:

Using $T_n = ar^{n-1}$

Given $T_3 = 9$ and $T_6 = 243$

Substituting $r = 3$ in equation (i)

$$ar^2 = 9 \quad \text{(i)}$$

$$9a = 9 \quad a = 9/9 \quad a = 1$$

$$ar^5 = 243 \quad \text{(ii)}$$

Divide equation(ii) by equation (i)

The first term = 1 and the common ratio = 3

$$Ar^5/ar^2 = 243/9 = 27$$

Ans (6mks)

$$R^3 = 3^3 \quad r = 3$$

Q13-(a) (i) Solution:

$$Y = X^{-1} + 5 \sin X$$

(ii) Solution:

$$u = 5x^3, du/dx = 15x^2, v = X+1, dv/dx = 1$$

$$\frac{dy}{dx} = -X^{-2} + 5\cos X$$

$$\text{Using, } \frac{dy}{dx} = vdu/dx - udv/dx/v^2$$

$$\frac{dy}{dx} = -1/X^2 + 5\cos X \text{ ans (3mks)}$$

$$= (X+1)(15X^2) - (5X^3)(1)/(X+1)^2$$

$$= 15X^3 + 15X^2 - 5X^3/(X+1)^2$$

$$\frac{dy}{dx} = 10X^3 + 15X^2/(X+1)^2 \text{ Ans (3mks)}$$

(b)-(i)solution:

(ii)-solution

$$\int (x - x^{-2}) dx = x^2/2 + x^{-1} + C$$

$$\int (3x + 1) dx = 3/x^2/2 + x + C \text{ ans (3mks)}$$

$$= x^2/2 + 1/x + C \text{ ans (3mks)}$$

Q14 Given $Z_1 = 6+2i$ and $Z_2 = 1+i$

(i)-solution

(ii)-solution

(iii)- solution $Z_1 Z_2 = (6+2i)(1+i)$

$$Z_1 + Z_2 = 6 + 2i + 1 + i$$

$$Z - Z = (6 + 2i) - (1 + i)$$

$$= 6 + 6i + 2i - 2$$

$$= 7+3i \text{ ans} (2\text{mks})$$

$$= 5+i \text{ ans} (2\text{mks})$$

$$= 4+8i \text{ ans} (3\text{mks})$$

(iv) Solution:

$$X=4, Y=8$$

$$|Z_1 Z_2| = \sqrt{x^2 + Y^2} = \sqrt{4^2 + 8^2} = \sqrt{16 + 64} = \sqrt{80} = \sqrt{16} \times \sqrt{5} = 4\sqrt{5} \text{ ans} (3\text{mks})$$

$$(v) \text{ solution } \tan \Theta = Y/X = 8/4 = 2$$

$$\text{Arg } Z_1 Z_2 = \tan^{-1}(2) \text{ ans.} (2\text{mks})$$

$$\text{Q15. (a) solution } Y = 2x^5 - x^2 - 3x + 5$$

$$\frac{dy}{dx} = 10x^4 + 2x^3 - 3$$

$$\frac{dy}{dx} = 10x^4 + \frac{2}{x^3} - 3 \text{ ans.} (4\text{mks})$$

$$(b) \text{ solution } Y + X^2 - 3x + 4$$

$$\frac{dy}{dx} = 2x - 3$$

$$\text{At } x = 1 \quad \frac{dy}{dx} = 2(1) - 3 = 2 - 3 = -1 \text{ ans} (4\text{mks})$$

$$(c) \text{ solution } Y = 3X^2$$

$$\frac{dy}{dx} = 6x = 6 \times 3 = 18$$

$$\text{Using } \frac{Y-Y_1}{X-X_1} = \frac{dy}{dx}$$

$$\text{Where } X_1 = 3, Y_1 = 27 \text{ and } \frac{dy}{dx} = 18$$

$$\frac{Y-27}{X-3} = 18 \quad Y-27 = 18(X-3)$$

$$Y - 27 = 18X - 54$$

$$18X + Y - 27 + 54 = 0 \quad \text{The equation of the tangent is } 18x + Y + 27 = 0 \text{ ans} (4\text{mks})$$

Q16 (a) solution

$$P = 25,000 \text{ SSP} \quad r = 18\% \quad t = 3 \frac{1}{2} \text{ years}$$

Using $I = Prt$ $I = 25,000 \times 18 \times 3.5/100$

$$I = 1575000/100$$

$$I = 15750 \text{ ans} \quad (4\text{mks})$$

(b) solution

$$P \propto 1/q \quad P = k/q \quad k = pq = 4 \times 5 = 20$$

$$P = 20/q \text{ at } q = 10 \quad p = 20/10 = 2 \quad P = 2 \text{ ans} \quad (4\text{mks})$$

(c) solution

Given $6 + 9 + 12 + \dots$

$$a = 6 \quad d = 9 - 6 = 3 \quad S_n = 132$$

$$\text{using } S_n = n/2 [2a + (n-1)d] \quad n/2 [2 \times 6 + 3(n-1)] = 132$$

$$n/2 [12 + 3n - 3] = 132 \quad n(9+3n) - 264 = 0$$

$$3n^2 + 9n - 264 = 0 \quad n^2 + 3n - 88 = 0$$

$$\text{Either } n + 11 = 0 \quad \text{or} \quad n - 8 = 0 \quad n = -11 \quad \text{or} \quad n = 8$$

The number of the terms of arithmetic is 8 terms ans (4mks)

Q17. (a) Solution $\int_1^2 (x^3 + x) dx$

$$= [x^4/4 + x^2/2]_1^2 = (2^4/4 + 2^2/2) - (1/4 + 1/2)$$

$$16/4 + 4/2 - (1+2)/4 = 16+8 - 3/4 = 21/4 \text{ ans} \quad (4\text{mks})$$

(b) Isolation $|b| = \sqrt{(-1)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \text{ units ans} \quad (2\text{mks})$

ii. solution

$$a + b = \begin{bmatrix} 5 \\ 6 \end{bmatrix} + \begin{bmatrix} -3 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 - 3 \\ 6 + 4 \end{bmatrix} = \begin{pmatrix} 2 \\ 10 \end{pmatrix}$$

$$\|a+b\| = \sqrt{2^2 + 10^2} = \sqrt{4 + 100} = \sqrt{104} \text{ units ans (2mks)}$$

(c) solution

$$\text{Given } \frac{dy}{dx} = 3 - 2x^2 \quad Y = \int (3 - 2x^2) dx$$

$$Y = 3x - 2/3X^3 + C \quad \text{at } X = 0, Y = 0, C = 0$$

The equation of the curve is $Y = 3x - 2/3X^3$ ans (4mks)